University of California, Berkeley
Physics H7C Fall 1999 (*Strovink*)

# EXAMINATION 2

**Directions.** Do all four problems (weights are indicated). This is a closed-book closed-note exam except for two $8\frac{1}{2} \times 11$ inch sheets containing any information you wish on both sides. You are free to approach the proctor to ask questions – but he or she will not give hints and will be obliged to write your question and its answer on the board. Calculators are allowed but not essential – roots, circular functions, *etc.*, may be left unevaluated if you do not know them. Use a bluebook. Do not use scratch paper – otherwise you risk losing part credit. Cross out rather than erase any work that you wish the grader to ignore. Justify what you do. Box or circle your answer.

**1.** (25 points)
The basis of scalar diffraction theory is the *Fresnel-Kirchoff integral formula*. In Fowles' notation (Eq. 5.11), this formula states

$$U_{\rm p} = -\frac{ikU_0 \exp\left(-i\omega t\right)}{4\pi} \times \\ \times \int\!\!\int \frac{\exp\big(ik(r+r')\big)}{rr'}\left[\hat{\mathbf{n}}\cdot\hat{\mathbf{r}} - \hat{\mathbf{n}}\cdot\hat{\mathbf{r}}'\right] d\mathcal{A}$$

where $\mathbf{r}'$ is a vector from the (point) source to a point on the aperture, $\mathbf{r}$ is a vector from the observer to the same point on the aperture,

$$U_0 \frac{\exp\big(i(kr' - \omega t)\big)}{r'}$$

is the optical disturbance at a point on the aperture, $U_{\rm p}$ is the optical disturbance at the observer, $\omega = ck = 2\pi c/\lambda$ is the angular frequency of the light, $d\mathcal{A}$ is an element of aperture area, and $\hat{\mathbf{n}}$ is the normal to $d\mathcal{A}$. [Note that, in a typical geometry (source on the left, aperture in the middle, observer on the right, and $\hat{\mathbf{n}}$ pointing to the left), $\hat{\mathbf{n}}\cdot\hat{\mathbf{r}}$ is positive while $\hat{\mathbf{n}}\cdot\hat{\mathbf{r}}'$ is negative, so that both terms in the square bracket are positive.]

Consider this simple geometry: Let $z$ be the axis pointing from left to right. Place the source at $(x, y, z) = (0, 0, -D)$, the observer at $(X, Y, D)$, and the aperture in the plane $z = 0$. The aperture is characterized by an *aperture function* $g(x, y)$ such that $g = 1$ where the aperture is open, and $g = 0$ where the aperture is opaque.

**a.** (10 points) Let $\delta$ be the maximum value of $\sqrt{x^2 + y^2}$ on the aperture plane for which the aperture is *not* opaque. Thus, for this part of the problem, there are three characteristic lengths: $\lambda$, $\delta$, and $D$. By moving around in the plane $z = D$, restricting her own coordinates $X, Y$ such that $\sqrt{X^2 + Y^2} \ll D$, the observer finds that the optical disturbance there is proportional to the *Fourier transform* of $g(x, y)$. As someone who understands the physics of diffraction, you realize that this information implies that a single strong condition must be satisfied which relates $\lambda$, $\delta$, and $D$. *Write down this condition.* (You needn't prove it, and you may omit factors of order unity.)

**b.** (15 points) For this part of the problem, take the aperture function to be

$$g(x, y) = 0, \quad x < 0$$
$$g(x, y) = 1, \quad x > 0\ .$$

This describes a "knife edge" at $x = 0$ extending from $y = -\infty$ to $y = \infty$. Therefore, in this part of the problem, $\delta = \infty$: the strong condition of part **a.** cannot be satisfied. In this part of the problem, the observer is fixed at $(0, 0, D)$, *i.e.* at $X = Y = 0$. With this aperture in place, the observer records an *irradiance* $I_{\rm a}$. With the aperture completely removed ($g \equiv 1$), the observer records an irradiance $I_0$. Give the ratio $I_{\rm a}/I_0$. To receive credit you must *explain why this ratio is correct.*

**2.** (25 points)
James Rainwater was awarded the Nobel Prize in the 1980's for experiments done at the Nevis (Columbia) cyclotron in the 1950's. He measured the sizes of nuclei using their interactions with muons (heavy electrons) which were in orbit about them.

In the following, use the Bohr picture to describe the muon orbit. For ease of numerical computation, you may take the natural length unit $\hbar/m_e c$ to be 400 fm; the ratio $m_\mu/m_e$ of muon to electron masses to be 200; and the fine structure constant $\alpha$ to be 1/150. You may neglect the difference between the muon's actual and reduced mass.

A muon in $n = 1$ Bohr orbit reacts with (is "captured" by) a $Z = 50$ nucleus before it decays:

$$\mu^- + (A, Z) \rightarrow (A, Z-1) + \nu_\mu,$$

where the neutrino $\nu_\mu$ has negligible rest mass. Assuming that the initial and final nuclei have the same infinitely large rest mass and therefore a negligible kinetic energy, what is the neutrino energy expressed in units of $m_e c^2$? (1% accuracy is sufficient.)

**3.** (25 points)
Consider the elastic scattering of a photon from an infinitely massive, perfectly reflective, spherical target of finite radius $R$ (like a bowling ball polished to a mirror finish). The bowling ball is centered on the origin. The photon is incident along the $\hat{\mathbf{z}}$ direction and scatters (reflects) into the direction $(\theta, \phi)$, where $\theta$ and $\phi$ are the usual spherical polar angles. Note that $\theta = 0$ means that the photon remains undeflected. For this problem, ignore diffraction and any other effects which arise from the wavelike properties of the photon.

**a.** (10 points) What is the total scattering cross section $\sigma_T$, corresponding to any deflection of the photon? (You don't need a calculation here, just a correct answer and a convincing explanation for it.)

**b.** (15 points) Calculate the differential cross section

$$\frac{d\sigma}{d\Omega},$$

where $d\Omega = \sin\theta \, d\theta \, d\phi$ is an element of solid angle. (When you integrate your result over the full solid angle, do you confirm your answer to **a.**?)

**4.** (25 points)
A nonrelativistic particle of mass $m$ is confined to a one-dimensional box extending from $x = 0$ to $x = L$. Here a "box" is a square potential well with infinite sides.

**a.** (10 points) In terms of $n$ and other constants, write down the energies $E_n$, $1 \leq n < \infty$, measured with respect to the bottom of the potential well, that the particle is allowed by Schrödinger's equation to have.

**b.** (15 points) Define $N(E)$ to be the total number of allowed states with energy $\leq E$. Taking $n \gg 1$, so that the distribution of $E$ is approximately continuous, calculate the density of states

$$\rho(E) \equiv \frac{dN}{dE}.$$